1.9 Stress, Strain, and Elasticity

A *rigid body* undergoes no deformation when acted upon by forces. This idealization will often serve as a good approximation for simplifying several problems in physics. In contrast, real bodies undergo some shape deformation when acted on by forces. A rigorous treatment of such deformations would take us outside the focus of this text and into the realm of continuum mechanics. In this section, however, we examine only the simplest examples of such situations to introduce a few important concepts used later in the text.

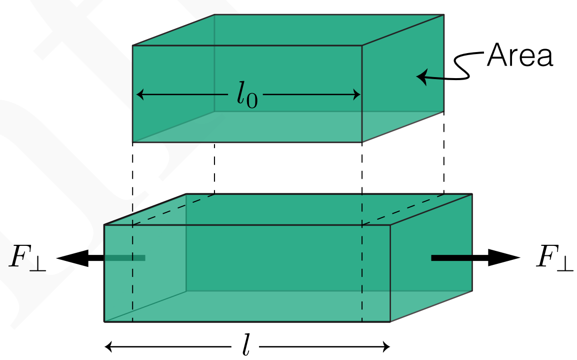
Stress is a physical quantity that expresses the forces that particles within a continuous body exert on each other across a boundary. *Strain* is the measure of the resulting deformation of the body. An *elastic* material deforms slightly under stress and returns to its original shape when the stress is removed. A *plastic* material deforms continuously and irreversibly above a critical stress and does not recover back to its original shape when the stress is removed. Here we consider only elastic deformations where stress and strain are linearly related with the elastic modulus as the proportionality.

1.9.1 Tensile and Compressive Stress and Strain

An object is under *tension* when forces of equal magnitude, $F_\perp$, but opposite in sign pull at opposite ends of an object. We define the tensile stress, σ, as the force per unit area,

$$\sigma_\perp = \frac{F_\perp}{A_0}, \tag{1.112}$$

where A_0 is the cross-sectional area through which the force is applied.



We define the tensile strain, ϵ_l as the deformation under the stress,

$$\epsilon_l = \frac{l - l_0}{l_0} = \frac{x}{l_0}. \tag{1.113}$$

where $x = l - l_0$. The *Young's modulus* is an elastic modulus given by the ratio of stress to strain,

$$Y = \frac{\sigma_\perp}{\epsilon_l} = \frac{F_\perp}{A}\frac{l_0}{x}. \tag{1.114}$$

Rearranging this expression, we obtain Hooke's law for a stretched rod or wire,

$$F_\perp = \underbrace{\frac{YA_0}{l_0}}_{-\kappa_f} x. \tag{1.115}$$

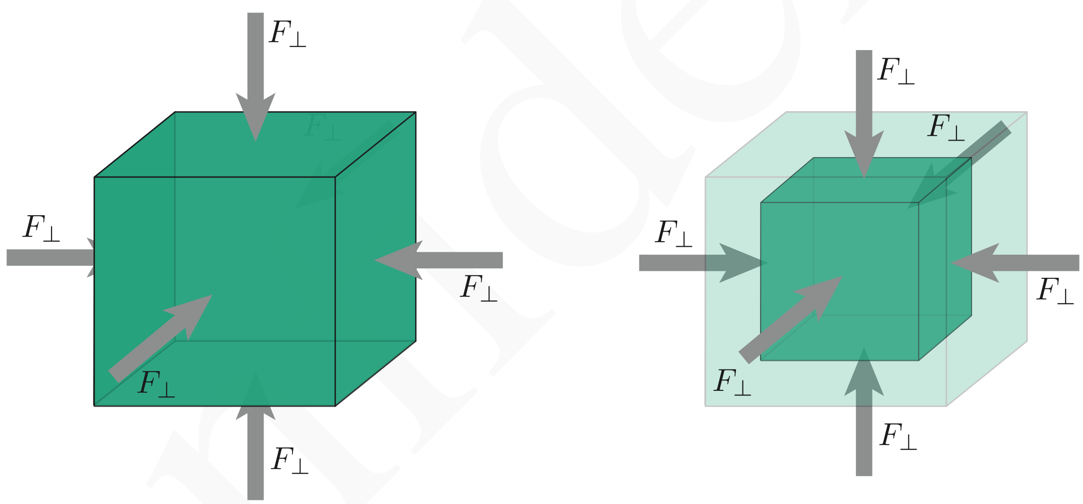
Similarly, an object is under *compression* when forces of equal magnitude, $F_\perp$, but opposite in sign push at opposite ends of the object. The compressive strain is defined

$$\epsilon_l = \frac{l_0 - l}{l_0} = \frac{x}{l_0}. \tag{1.116}$$

and the Eqs (1.114) and (1.115) still hold.

1.9.2 Bulk Stress and Strain

When a uniform pressure, p, is applied to all sides, an object is under *bulk stress*.



The change in pressure is the force per unit area,

$$\sigma_\perp = \frac{F_\perp}{A} = \Delta p. \tag{1.117}$$

The bulk strain is

$$\epsilon_V = \frac{V - V_0}{V_0} = \frac{\Delta V}{V_0}. \tag{1.118}$$

The bulk modulus is an elastic modulus that is given by

$$B = \frac{\sigma_\perp}{\epsilon_V} = V_0 \frac{dp}{dV}. \tag{1.119}$$

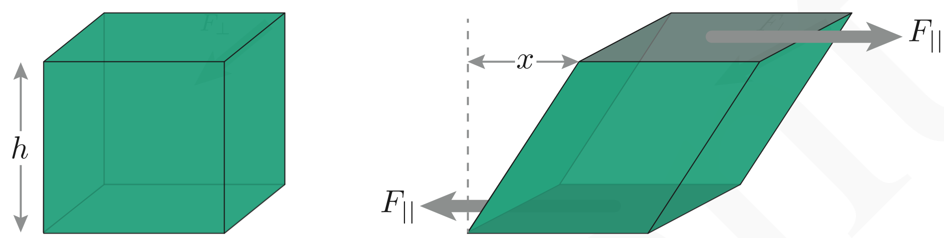
1.9.3 Shear Stress and Strain

An object is under *shear stress* when forces of equal magnitude, $F_{||}$, but opposite in sign pull at opposite ends of an object. We define the shear stress, $\sigma_{\parallel}$, as the force per unit area,

$$\sigma_{\parallel} = \frac{F_{\parallel}}{A_0}. \tag{1.120}$$

We define the shear strain, ϵ_s as the deformation under the stress,

$$\epsilon_s = \frac{x}{h} \tag{1.121}$$



The *shear modulus* is an elastic modulus given by

$$G = \frac{\sigma_{\parallel}}{\epsilon_s} = \frac{F_{\parallel}}{A_0}\frac{h}{x}. \tag{1.122}$$

The concepts of shear stress, shear strain, and shear modulus apply only to solids. Liquids and gases do not have a definite shape and cannot support a shear stress.