

## Fixed Field Gradient NMR Diffusion Measurements Using Bessel Function Fits to the Spin-Echo Signal

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A new technique for diffusion measurement by NMR using a fixed field gradient is described. The method makes use of a Bessel function fit analysis to the spin-echo signal in order to determine both the applied gradient and the diffusion coefficient simultaneously. Results of tests of the method using water and benzene are presented, and the utility of the technique is demonstrated with a report of preliminary results for a determination of the diffusion of naphthalene dissolved in compressed supercritical carbon dioxide. © 1987 Academic Press, Inc.

### INTRODUCTION

Nuclear magnetic resonance techniques for measuring translational diffusion are well known. In 1954, Carr and Purcell (1) expanded on a suggestion by Hahn (2) and showed that by using the NMR spin echo and an applied magnetic gradient molecular diffusion could be followed by "labeling" molecules using their nuclear magnetic moments. Since that time, the fixed field linear magnetic gradient spin-echo method has been used by many investigators to measure the diffusion coefficient,  $D$ , in a wide variety of systems.

Before the technique can yield diffusion information, however, it has one important requirement—an exact measurement of the applied linear magnetic gradient  $G$  must be obtained. This determination of the applied gradient has usually been made in one of three ways: by theoretical calculation, by use of a standard sample of known diffusion coefficient, or by an analysis of the NMR free induction decay. Each method has its drawbacks. A theoretical calculation of the gradient field from the coil dimensions is difficult and may be experimentally inaccurate. The use of a standard with a known diffusion coefficient to determine  $G$  has the problem of finding a suitable reference sample. This is further complicated by the lack of consistency in the published values of diffusion coefficients (see, for example, Weingartner's review on the diffusion of water at 25°C (3)). However, as Harris *et al.* (4) have shown, this method can be used to obtain diffusion data to good accuracy ( $\pm 2\%$ ), and the method has often been used in the literature. We have used this method in our laboratory as well (5) and have obtained reasonable results. Unfortunately, the use of a standard sample to calibrate  $G$  can be inconvenient and difficult in our laboratory when we use our NMR apparatus for measurements under extreme conditions of temperature and pressure (6). If accurate diffusion data for the sample of interest under ambient conditions do not exist, we must first calibrate  $G$  with a suitable standard, dismantle the system, change samples,

and reseal the pressure vessel—a process that can easily introduce error into the gradient calibration.

The third method for calibration of the gradient was proposed in the original paper by Carr and Purcell (1). This method utilizes the time dependence of the magnetization for a cylindrical sample in the presence of a linear magnetic gradient perpendicular to the cylinder axis, given by

$$M(t) = M_0[2J_1(\gamma Grt)/(\gamma Grt)], \quad [1]$$

where  $\gamma$  is the gyromagnetic ratio of the observed nucleus,  $G$  is the applied gradient,  $r$  is the sample radius, and  $J_1$  signifies the first-order Bessel function. This method of gradient calibration has the advantage of giving a direct determination of  $G$  and has also been used extensively in the literature. The most common approach has been to make use of the known zeros of  $J_1(x)$  and to compare them to the experimental free induction decay in the presence of the gradient to calculate  $G$ . This calculation is prone to many systematic errors, however, which have been thoroughly discussed by Murday (7). Another approach to this method has been described by Hrovat and Wade (8), who use the off-resonance echo envelope over a relative intensity of 0.97 to 0.17 and a polynomial approximation to the inverse lineshape to obtain  $G$ . Hrovat and Wade have also discussed possible errors in the determination (9), expanding on Murday's discussion (7). Yet another approach has been pursued by Fukushima *et al.* (10), where they Fourier transform the spin echo to obtain the field distribution across the sample in the frequency domain. This one-dimensional projection of the sample is then used to obtain  $G$  using the known radius of the sample cell. Another similar analysis has recently been discussed by Hall *et al.* (11).

In contrast to the above methods, the most straightforward approach for the use of Eq. [1] to determine the gradient would be an actual Bessel function computer fit to the entire spin-echo signal. This has been previously proposed by several authors (7, 10) but to our knowledge no results for such an analysis have been reported in the literature. In this paper we present our results for the use of Bessel function fits to the spin-echo signal to perform fixed field gradient NMR diffusion measurements. The great advantage of this approach is that through the spin-echo fit we simultaneously obtain information on both the applied gradient and the effects of diffusion. Thus, no independent calibration of the applied magnetic gradient is required—a direct determination of the magnitude of the gradient is obtained in the process of the diffusion measurement.

In the next section of this paper we will describe the experimental method in detail. Following that discussion, the results obtained for tests of the technique using water and benzene will be given. Finally, the utility of the technique will be demonstrated with a report of preliminary results from a study of the diffusion of solid naphthalene dissolved in compressed supercritical carbon dioxide.

## EXPERIMENTAL

*Apparatus.* These experiments were performed using a 14.1 kG electromagnet with a wide gap (3.8 in.) that accommodates the high-temperature high-pressure NMR probe and pressure vessel which have been described in detail elsewhere (12). The

temperature was controlled to within  $\pm 0.2^\circ\text{C}$  as monitored by a copper-constantan thermocouple located slightly above the sample region. For the supercritical experiments, carbon dioxide pressure was generated with a 30,000 psi gas compressor (Superpressure, Inc.) and measured to an accuracy of  $\pm 2$  bar with a Heise-Bourdon pressure gauge (Dresser Industries).

The homemade NMR spectrometer and pulse programmer, similar to those described previously (5), are automated with an LSI-11 computer (Digital Equipment Corp.). The computer is interfaced to a Nicolet Explorer digital oscilloscope for acquisition of data. The software that coordinates the data acquisition process as well as the subsequent data reduction is written in FORTRAN-77 and LSI-11 Assembly Language. Detailed schematics and software are available from the authors upon request.

The linear magnetic gradient was applied using anti-Helmholtz coils which are located outside of the cooling jacket of the high-temperature high-pressure probe assembly. The diameter of the coils (4.4 in.) was chosen to provide the largest region of uniform gradient ( $I$ ) given the required separation distance. Currents to 3 A were supplied to the coils with a Power Designs, Inc., DC power supply which produced a gradient field at the sample (determined as described below) of approximately 2.5 G/cm.

In order to have an accurate value of the sample radius, commercial high-precision sample tubes (Wilmad, 8 mm o.d.) were used in the tests with water and benzene. The naphthalene-supercritical carbon dioxide experiments used a sample cell described previously (13), which is machined from a high-temperature polyimide plastic (VespeI, Dupont Co.) to 0.250 in. inner diameter.

*Diffusion measurement.* For the  $90^\circ\text{-}\tau\text{-}180^\circ$  spin-echo fixed field gradient NMR diffusion measurement, the amplitude of the echo maximum  $A$  is given by

$$A(2\tau, G, D) = A_0 \exp\left[-\frac{2}{3} \gamma^2 G^2 D \tau^3 - (2\tau/T_2)\right], \quad [2]$$

where  $G$  is the linear field gradient,  $D$  is the diffusion coefficient,  $T_2$  is the transverse relaxation time, and  $\tau$  is the time of pulse separation. The spin-echo shape is given by Eq. [1], which we now express as

$$A(t - t_{\max}) = A(t_{\max}) \left( \frac{2J_1[\gamma G r(t - t_{\max})]}{[\gamma G r(t - t_{\max})]} \right), \quad [3]$$

where  $t_{\max}$  is the time of the echo maximum and the other symbols are as previously described. Our method for diffusion measurement makes use of Eqs. [2] and [3] to obtain both  $D$  and  $G$  simultaneously with one experiment.

The diffusion experiment begins with a choice of appropriate pulse separation  $\tau$  given the  $T_2$  and diffusion characteristics of the sample. With  $\tau$  held constant, spin echoes are acquired while systematically varying the current supplied to the gradient coils. Usually echoes are acquired for 8–10 different current settings, resulting in an echo signal attenuation of about 70% at the largest gradients. Two conditions must be met during data acquisition. First, all spin echoes are acquired on resonance; if necessary, the resonance position is reset for each value of the applied gradient to account for any displacement of the center of the sample from the position at which

the applied gradient contribution is zero (7). Our setup usually required a slight  $H_0$  field adjustment to bring the sample back on resonance after energizing the gradient coils. Second, digitizer dwell times are chosen to assure adequate representation of the echo shape for all values of the applied gradient. We used dwell times of 2–20  $\mu$ s.

The data analysis then begins with a computer fit of each baseline-corrected spin echo to Eq. [3]. The software is based on a Bessel function look-up table which was generated on a Digital Equipment Corporation VAX-11 computer using an IMSL library routine (International Mathematical and Statistical Libraries, Inc.). The fit is performed using a least-squares grid search routine (14) with  $t_{\max}$ ,  $A(t_{\max})$ , and  $G$  as the adjustable parameters. Typically 150 equally spaced points from the digitized spin echo covering a range including five zeros in  $J_1(x)/x$  on each side of the echo are used in the fit. An example of a Bessel function spin-echo fit is shown in Fig. 1.

We found spin-echo fits to be a very accurate and advantageous method for gradient calibration. Many of the problems inherent in the method to determine  $G$  by use of the spin-echo zeros (7) are circumvented with our technique. For example, timing delays are accounted for by the use of  $t_{\max}$  as an adjustable parameter in the fit using Eq. [3]. Also, resonance position is easily checked in the fit by noting whether the echo shape is symmetrical. Another problem which our method monitors is any inhomogeneity in  $G$ . If the applied gradient is not homogeneous, the spin-echo shape is affected and a poor Bessel function fit is obtained.

As suggested by Fukushima *et al.* (10), this can be further confirmed by performing a Fourier transform of half the spin echo to obtain the one-dimensional projection of the sample. Any inhomogeneity in  $G$  will distort the expected semiellipse image of the sample. For example, we found experimentally that if we located the thermocouple too close to the sample region, the frequency domain image of the sample became

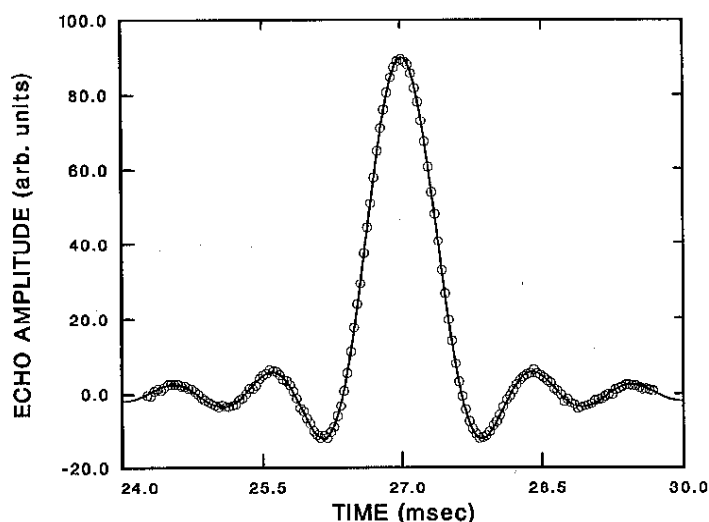


FIG. 1. Spin-echo Bessel function fit analysis for a water sample in an 8 mm sample tube. The circles are the experimental digitized spin echo; the solid line is the best fit using Eq. [3]. The analysis gives a value of  $G = 0.646$  G/cm for the applied linear magnetic gradient (for details see text).

noticeably distorted, which corresponded to a poor Bessel function fit to the spin-echo shape in the time domain. A final problem that can be accounted for with our method is that of background gradients. Since the Bessel function fit gives the actual gradient at the sample, any background gradient inherent in the magnetic field that contributes to the applied gradient is automatically included in the determination of  $G$ .

As is shown in Fig. 1 for a water sample in an 8 mm sample tube, excellent agreement is obtained between the computer fit to the theoretical shape given by Eq. [3] and the digitized experimental spin-echo signal. In cases with good signal-to-noise ratios, we obtained a reproducibility of  $\pm 1\%$  in replicate calibrations of the gradient using this analysis. We believe these results show that Bessel function fits to the spin echo can be very accurate techniques to determine the applied gradient.

The calculation of the diffusion coefficient follows directly from the Bessel fits to the spin echoes. The computer fit to each echo acquired in a series of applied gradients yields the value of the gradient  $G$  and the amplitude of the echo maximum  $A(t_{\max})$ . These data are then used to obtain the diffusion coefficient by a nonlinear least-squares fit to

$$A(t_{\max}) = A'_0 \exp[-\frac{2}{3}\gamma^2 G^2 D \tau^3], \quad [4]$$

where the constant  $A'_0$  now includes  $T_2$  effects (and can include other  $\tau$ -dependent effects as discussed below). An example of a spin-echo series with increasing  $G$  is shown in the stacked plot of Fig. 2. This figure shows that all the information necessary for a diffusion measurement is available in this one series of echoes: the width of the echoes gives a measure of the applied gradient while the attenuation of the echo amplitudes allows a measure of the diffusion of the sample. Thus, this technique using Bessel function fits to the spin-echo signal provides a self-contained method for the measurement of diffusion by NMR—gradient calibration and diffusion measurement are performed simultaneously. In the next section we give some results using the method.

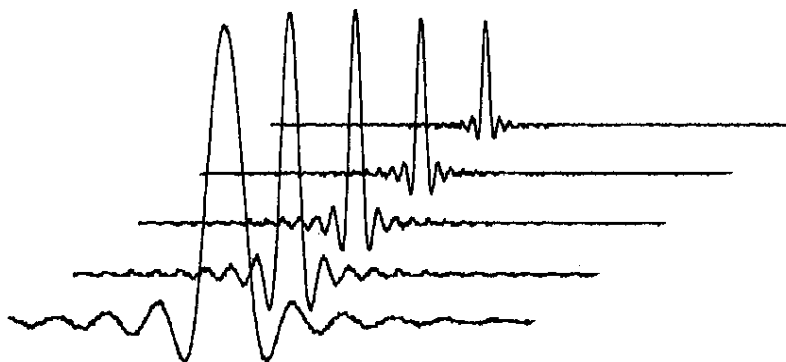


FIG. 2. Spin-echo series with increasing  $G$  for  $\tau = 0.027$  s using a water sample at  $27.2^\circ\text{C}$ . The series contains information on both the applied gradient (Bessel function echo shape) and the effects of diffusion (echo attenuation). Analysis gives the values of the gradient  $G = 0.430, 0.860, 1.29, 1.72,$  and  $2.15$  G/cm and a self-diffusion coefficient of  $D = 2.47 \times 10^{-5}$   $\text{cm}^2/\text{s}$  (for details see text).

## RESULTS

*Diffusion tests using water and benzene.* In order to assess the precision and accuracy of our diffusion measurement technique we have performed tests using water and benzene. These liquids have self-diffusion coefficients which have been accurately determined by tracer diffusion studies (15, 16). Distilled, deionized water and reagent-grade benzene (J. T. Baker Co.) were used. The tests were performed in our high-temperature high-pressure NMR probe at 1 bar. The results of the diffusion measurements using the spin-echo Bessel function fit technique are shown in Table 1. Each reported self-diffusion coefficient is the average of three to four replicate determinations which had a reproducibility of approximately 2%.

As can be seen in the table, with our method we were able to obtain the literature values to within 2.5% for the self-diffusion coefficients of water and benzene at various temperatures. These data show the excellent accuracy that can be obtained for diffusion data using the spin-echo Bessel function fit technique; we would estimate the overall accuracy of the method to be  $\pm 3\%$ . As a comparison, in our previous paper (5) describing diffusion measurements using the spin-echo zeros to calibrate the gradient, we could only obtain an accuracy of approximately  $\pm 8\%$ . The new spin-echo Bessel fit method therefore affords a substantial improvement over previous techniques. Finally, the accuracy of our new technique compares favorably with the results obtained by Harris *et al.* (4), where an independent calibration of the gradient coils is required.

*Naphthalene diffusion in supercritical carbon dioxide.* In order to illustrate the utility of the spin-echo Bessel fit technique, we now report the results for preliminary measurements of the diffusion of solid naphthalene dissolved in compressed supercritical carbon dioxide. This work is part of our overall studies of the properties of supercritical fluids and their mixtures (17).

We have measured the diffusion of naphthalene dissolved in  $\text{CO}_2$  at 35 and 55°C and pressures from 80 to 1000 bar. Under these conditions of temperature and pressure, the phase behavior of this class III mixture (18) is such that the only phases present are solid naphthalene and a supercritical phase consisting of naphthalene dissolved in  $\text{CO}_2$ . In a previous study (13) we determined the equilibrium solubilities of the solid

TABLE 1

Experimental Results for Diffusion Measurement Tests  
of Spin-Echo Bessel Function Fit Technique

Sample (°C)	$D_{\text{exp}} \times 10^5$ ( $\text{cm}^2/\text{s}$ )	$D_{\text{lit}} \times 10^5$ <sup>a</sup> ( $\text{cm}^2/\text{s}$ )	$\Delta\%$ <sup>b</sup>
Water at 27.2	2.47	2.43	1.6
31.9	2.65	2.71	2.2
40.4	3.34	3.26	2.5
Benzene at 27.5	2.35	2.31	1.7

<sup>a</sup> Literature values interpolated from quadratic fits to  $D = f(T)$  diffusion data in Refs. (15) and (16).

<sup>b</sup> Percentage relative deviation =  $100|D_{\text{exp}} - D_{\text{lit}}|/D_{\text{lit}}$ .

naphthalene-supercritical CO<sub>2</sub> solution. We now present results for the first *in situ* NMR measurement of the diffusion of the dissolved naphthalene in solution.

The use of the spin-echo Bessel fit technique was important for these measurements in two respects. First, no reliable diffusion data exist for this system to use as reference with which to perform a gradient calibration. But as we have described, the Bessel fit technique requires no independent gradient calibration. Second, the two types of magnetically inequivalent naphthalene protons are strongly spin-spin coupled and thus exhibit a complex pattern of echo amplitude J-modulation (19). This complicates any NMR diffusion measurement in which the pulse separation is varied at a fixed value of the gradient. In our Bessel fit technique, however, we hold the pulse separation  $\tau$  constant while varying  $G$ . This allows us to choose an appropriate  $\tau$  to optimize the echo amplitude in the presence of the J-modulation. Any effects of the echo modulation are then included in the constant  $A_0$  of Eq. [4] when calculating the diffusion coefficient.

The results for our investigation of the diffusion of naphthalene dissolved in supercritical CO<sub>2</sub> are shown in Fig. 3. The naphthalene diffusion data are easily interpreted by considering the effects of the density of CO<sub>2</sub> under these conditions. At low pressures and lesser densities of CO<sub>2</sub> the naphthalene diffusion is greater and a significant temperature dependence is observed. At higher pressures and greater CO<sub>2</sub> densities, diffusion slows and shows only a small dependence on temperature and pressure.

A detailed analysis of the diffusion data will be presented elsewhere. These preliminary results show that the spin-echo Bessel fit technique can be advantageously applied to these systems.

#### CONCLUSION

We have presented a new technique to measure diffusion by NMR which makes use of a first-order Bessel function fit analysis to the spin-echo signal. We have shown

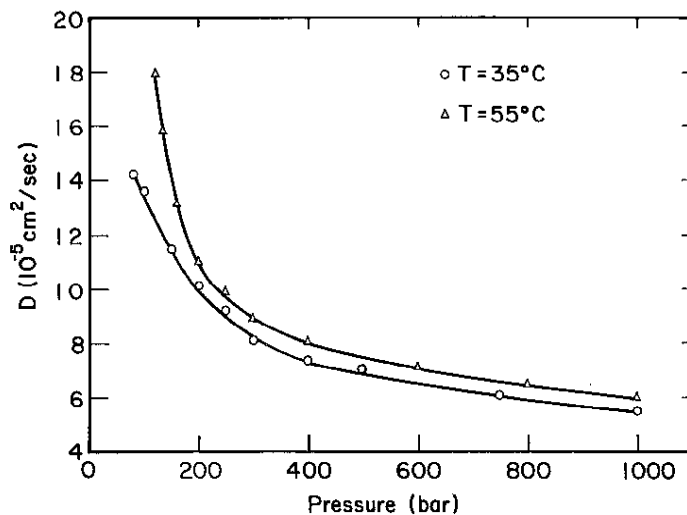


FIG. 3. Experimental diffusion coefficients for naphthalene dissolved in carbon dioxide as a function of temperature and pressure for the supercritical solution.

that the method can obtain diffusion data to an accuracy of better than  $\pm 3\%$ . The advantages of the new technique are (i) no independent experiment to calibrate the applied magnetic gradient is required, (ii) the spin-echo fit provides a direct determination of  $G$  that is significantly more accurate than other methods to determine the gradient, and (iii) gradient calibration and diffusion measurement are performed simultaneously on the sample of interest under the desired conditions.

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